Theorem 1 *If $i \geq 1$ then $\Pi_i = \Sigma_i$ implies $\Pi_{i+1} = \Sigma_{i+1}$, therefore the polynomial hierarchy collapses to level i.*

Proof: Let $L \in \Sigma_{i+1}$, we prove that $L \in \Sigma_i$. The proof for Π is analogous.
There exists a polynomial relation R:

$$x \in L \iff \exists y_1 \forall y_2 \exists ... Q y_{i+1} R(x, y_1, y_2, ..., y_{i+1})$$

Consequently, there exists an $L' \in \Pi_i$ such that

$$x \in L \iff \exists y_1 : (x, y_1) \in L'.$$

But $\Pi_i = \Sigma_i$, so $L' \in \Sigma_i$, that is, there exists a polynomial relation S:

$$x \in L \iff \exists y_1 \exists y_2 \forall y_3 \exists ... Q y_{i+1} S(x, y_1, y_2, ..., y_{i+1})$$

equivalently:

$$x \in L \iff \exists (y_1, y_2) \forall y_3 \exists ... Q y_{i+1} S(x, (y_1, y_2), ..., y_{i+1}),$$

therefore L is in Σ_i. Qed.